

# Mathematics 9709 (As and A level)

## 5. Syllabus content

The mathematical content for each unit in the scheme is detailed below. The order in which topics are listed is not intended to imply anything about the order in which they might be taught.

As well as demonstrating skill in the appropriate techniques, candidates will be expected to apply their knowledge in the solution of problems. Individual questions set may involve ideas and methods from more than one section of the relevant content list.

For all units, knowledge of the content of Cambridge O Level/Cambridge IGCSE Mathematics is assumed. Candidates will be expected to be familiar with scientific notation for the expression of compound units, e.g.  $5 \text{ m s}^{-1}$  for 5 metres per second.

Unit P1: Pure Mathematics 1 (Paper 1)	
	<i>Candidates should be able to:</i>
<b>1. Quadratics</b>	<ul style="list-style-type: none"><li>• carry out the process of completing the square for a quadratic polynomial <math>ax^2 + bx + c</math> and use this form, e.g. to locate the vertex of the graph of <math>y = ax^2 + bx + c</math> or to sketch the graph</li><li>• find the discriminant of a quadratic polynomial <math>ax^2 + bx + c</math> and use the discriminant, e.g. to determine the number of real roots of the equation <math>ax^2 + bx + c = 0</math></li><li>• solve quadratic equations, and linear and quadratic inequalities, in one unknown</li><li>• solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic</li><li>• recognise and solve equations in <math>x</math> which are quadratic in some function of <math>x</math>, e.g. <math>x^4 - 5x^2 + 4 = 0</math>.</li></ul>
<b>2. Functions</b>	<ul style="list-style-type: none"><li>• understand the terms function, domain, range, one-one function, inverse function and composition of functions</li><li>• identify the range of a given function in simple cases, and find the composition of two given functions</li><li>• determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases</li><li>• illustrate in graphical terms the relation between a one-one function and its inverse.</li></ul>

<p><b>3. Coordinate geometry</b></p>	<ul style="list-style-type: none"> <li>• find the length, gradient and mid-point of a line segment, given the coordinates of the end-points</li> <li>• find the equation of a straight line given sufficient information (e.g. the coordinates of two points on it, or one point on it and its gradient)</li> <li>• understand and use the relationships between the gradients of parallel and perpendicular lines</li> <li>• interpret and use linear equations, particularly the forms <math>y = mx + c</math> and <math>y - y_1 = m(x - x_1)</math></li> <li>• understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations (including, in simple cases, the correspondence between a line being tangent to a curve and a repeated root of an equation).</li> </ul>
<p><b>4. Circular measure</b></p>	<ul style="list-style-type: none"> <li>• understand the definition of a radian, and use the relationship between radians and degrees</li> <li>• use the formulae <math>s = r\theta</math> and <math>A = \frac{1}{2}r^2\theta</math> in solving problems concerning the arc length and sector area of a circle.</li> </ul>
<p><b>5. Trigonometry</b></p>	<ul style="list-style-type: none"> <li>• sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians)</li> <li>• use the exact values of the sine, cosine and tangent of <math>30^\circ</math>, <math>45^\circ</math>, <math>60^\circ</math>, and related angles, e.g. <math>\cos 150^\circ = -\frac{1}{2}\sqrt{3}</math></li> <li>• use the notations <math>\sin^{-1}x</math>, <math>\cos^{-1}x</math>, <math>\tan^{-1}x</math> to denote the principal values of the inverse trigonometric relations</li> <li>• use the identities <math>\frac{\sin i}{\cos i} = \tan i</math> and <math>\sin^2 i + \cos^2 i = 1</math></li> <li>• find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included).</li> </ul>
<p><b>6. Vectors</b></p>	<div style="display: flex; align-items: center; margin-bottom: 10px;"> <div style="margin-right: 20px;"> <p>use standard notations for vectors, i.e.</p> <p><math>\vec{AB}</math>, <math>\mathbf{a}</math></p> </div> <div style="margin-right: 20px;"> <p><math>x</math></p> <p><math>y</math></p> </div> <div style="margin-right: 20px;"> <p><math>\dots</math></p> </div> <div style="margin-right: 20px;"> <p><math>x</math></p> <p><math>y</math></p> <p><math>z</math></p> </div> <div> <p>, <math>x\mathbf{i} + y\mathbf{j} + z\mathbf{k}</math>,</p> </div> </div> <ul style="list-style-type: none"> <li>• carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms</li> <li>• use unit vectors, displacement vectors and position vectors</li> <li>• calculate the magnitude of a vector and the scalar product of two vectors</li> <li>• use the scalar product to determine the angle between two directions and to solve problems concerning perpendicularity of vectors.</li> </ul>

<p><b>7. Series</b></p>	<ul style="list-style-type: none"> <li>• use the expansion of <math>(a + b)^n</math>, where <math>n</math> is a positive integer (knowledge of the greatest term and properties of the coefficients are not required, but the notations <math>\sum_{r=0}^n \binom{n}{r} a^{n-r} b^r</math> and <math>n!</math> should be known)</li> <li>• recognise arithmetic and geometric progressions</li> <li>• use the formulae for the <math>n</math>th term and for the sum of the first <math>n</math> terms to solve problems involving arithmetic or geometric progressions</li> <li>• use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression.</li> </ul>
<p><b>8. Differentiation</b></p>	<ul style="list-style-type: none"> <li>• understand the idea of the gradient of a curve, and use the notations <math>f(x)</math>, <math>f'(x)</math>, <math>\frac{dy}{dx}</math> and <math>\frac{d^2y}{dx^2}</math> (the technique of differentiation from first principles is not required)</li> <li>• use the derivative of <math>x^n</math> (for any rational <math>n</math>), together with constant multiples, sums, differences of functions, and of composite functions using the chain rule</li> <li>• apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change (including connected rates of change)</li> <li>• locate stationary points, and use information about stationary points in sketching graphs (the ability to distinguish between maximum points and minimum points is required, but identification of points of inflexion is not included).</li> </ul>
<p><b>9. Integration</b></p>	<ul style="list-style-type: none"> <li>• understand integration as the reverse process of differentiation, and integrate <math>(ax + b)^n</math> (for any rational <math>n</math> except <math>-1</math>), together with constant multiples, sums and differences</li> <li>• solve problems involving the evaluation of a constant of integration, e.g. to find the equation of the curve through <math>(1, -2)</math> for which <math>\frac{dy}{dx} = 2x + 1</math></li> <li>• evaluate definite integrals (including simple cases of 'improper' integrals, such as <math>\int_0^1 x^{-\frac{1}{2}} dx</math> and <math>\int_1^3 x^{-2} dx</math>)</li> <li>• use definite integration to find: <ul style="list-style-type: none"> <li>– the area of a region bounded by a curve and lines parallel to the axes, or between two curves</li> <li>– a volume of revolution about one of the axes.</li> </ul> </li> </ul>

**Unit P2: Pure Mathematics 2 (Paper 2)**

Knowledge of the content of unit P1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

	<i>Candidates should be able to:</i>
<b>1. Algebra</b>	<ul style="list-style-type: none"> <li>understand the meaning of <math>x</math>, and use relations such as <math>a = b \Leftrightarrow a^2 = b^2</math> and <math>x - a &lt; b \Leftrightarrow a - b &lt; x &lt; a + b</math> in the course of solving equations and inequalities</li> <li>divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)</li> <li>use the factor theorem and the remainder theorem, e.g. to find factors, solve polynomial equations or evaluate unknown coefficients.</li> </ul>
<b>2. Logarithmic and exponential functions</b>	<ul style="list-style-type: none"> <li>understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)</li> <li>understand the definition and properties of <math>e^x</math> and <math>\ln x</math>, including their relationship as inverse functions and their graphs</li> <li>use logarithms to solve equations of the form <math>a^x = b</math>, and similar inequalities</li> <li>use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.</li> </ul>
<b>3. Trigonometry</b>	<ul style="list-style-type: none"> <li>understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude</li> <li>use trigonometrical identities for the simplification and exact evaluation of expressions and, in the course of solving equations, select an identity or identities appropriate to the context, showing familiarity in particular with the use of: <ul style="list-style-type: none"> <li><math>\sec^2 i \equiv 1 + \tan^2 i</math> and <math>\operatorname{cosec}^2 i \equiv 1 + \cot^2 i</math></li> <li>the expansions of <math>\sin(A \pm B)</math>, <math>\cos(A \pm B)</math> and <math>\tan(A \pm B)</math></li> <li>the formulae for <math>\sin 2A</math>, <math>\cos 2A</math> and <math>\tan 2A</math></li> <li>the expressions of <math>a \sin i + b \cos i</math> in the forms <math>R \sin(i + \alpha)</math> and <math>R \cos(i - \alpha)</math>.</li> </ul> </li> </ul>
<b>4. Differentiation</b>	<ul style="list-style-type: none"> <li>use the derivatives of <math>e^x</math>, <math>\ln x</math>, <math>\sin x</math>, <math>\cos x</math>, <math>\tan x</math>, together with constant multiples, sums, differences and composites</li> <li>differentiate products and quotients</li> <li>find and use the first derivative of a function which is defined parametrically or implicitly.</li> </ul>

<p><b>5. Integration</b></p>	<ul style="list-style-type: none"> <li>• extend the idea of 'reverse differentiation' to include the integration of <math>e^{ax+b}</math>, <math>\frac{1}{ax+b}</math>, <math>\sin(ax+b)</math>, <math>\cos(ax+b)</math> and <math>\sec^2(ax+b)</math> (knowledge of the general method of integration by substitution is not required)</li> <li>• use trigonometrical relationships (such as double-angle formulae) to facilitate the integration of functions such as <math>\cos^2 x</math></li> <li>• use the trapezium rule to estimate the value of a definite integral, and use sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate.</li> </ul>
<p><b>6. Numerical solution of equations</b></p>	<ul style="list-style-type: none"> <li>• locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change</li> <li>• understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation</li> <li>• understand how a given simple iterative formula of the form <math>x_{n+1} = F(x_n)</math> relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy (knowledge of the condition for convergence is not included, but candidates should understand that an iteration may fail to converge).</li> </ul>

**Unit P3: Pure Mathematics 3 (Paper 3)**

Knowledge of the content of unit P1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

	Candidates should be able to:
<p><b>1. Algebra</b></p>	<ul style="list-style-type: none"> <li>• understand the meaning of <math> x </math>, and use relations such as <math>a = b \Leftrightarrow a^2 = b^2</math> and <math>x - a &lt; b \Leftrightarrow a - b &lt; x &lt; a + b</math> in the course of solving equations and inequalities</li> <li>• divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)</li> <li>• use the factor theorem and the remainder theorem, e.g. to find factors, solve polynomial equations or evaluate unknown coefficients</li> <li>• recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than:               <ul style="list-style-type: none"> <li>– <math>(ax + b)(cx + d)(ex + f)</math></li> <li>– <math>(ax + b)(cx + d)^2</math></li> <li>– <math>(ax + b)(x^2 + c^2)</math></li> </ul>               and where the degree of the numerator does not exceed that of the denominator             </li> <li>• use the expansion of <math>(1 + x)^n</math>, where <math>n</math> is a rational number and <math> x  &lt; 1</math> (finding a general term is not included, but adapting the standard series to expand e.g. <math>Kx^2</math> is included).</li> </ul>
<p><b>2. Logarithmic and exponential functions</b></p>	<ul style="list-style-type: none"> <li>• understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)</li> <li>• understand the definition and properties of <math>e^x</math> and <math>\ln x</math>, including their relationship as inverse functions and their graphs</li> <li>• use logarithms to solve equations of the form <math>a^x = b</math>, and similar inequalities</li> <li>• use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.</li> </ul>

<p><b>3. Trigonometry</b></p>	<ul style="list-style-type: none"> <li>understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude</li> <li>use trigonometrical identities for the simplification and exact evaluation of expressions and, in the course of solving equations, select an identity or identities appropriate to the context, showing familiarity in particular with the use of: <ul style="list-style-type: none"> <li><math>\sec^2 i \equiv 1 + \tan^2 i</math> and <math>\operatorname{cosec}^2 i \equiv 1 + \cot^2 i</math></li> <li>the expansions of <math>\sin(A \pm B)</math>, <math>\cos(A \pm B)</math> and <math>\tan(A \pm B)</math></li> <li>the formulae for <math>\sin 2A</math>, <math>\cos 2A</math> and <math>\tan 2A</math></li> <li>the expressions of <math>a \sin i + b \cos i</math> in the forms <math>R \sin(i \pm \alpha)</math> and <math>R \cos(i \pm \alpha)</math>.</li> </ul> </li> </ul>
<p><b>4. Differentiation</b></p>	<ul style="list-style-type: none"> <li>use the derivatives of <math>e^x</math>, <math>\ln x</math>, <math>\sin x</math>, <math>\cos x</math>, <math>\tan x</math>, together with constant multiples, sums, differences and composites</li> <li>differentiate products and quotients</li> <li>find and use the first derivative of a function which is defined parametrically or implicitly.</li> </ul>
<p><b>5. Integration</b></p>	<ul style="list-style-type: none"> <li>extend the idea of 'reverse differentiation' to include the integration of <math>e^{ax+b}</math>, <math>\frac{1}{ax+b}</math>, <math>\sin(ax+b)</math>, <math>\cos(ax+b)</math> and <math>\sec^2(ax+b)</math></li> <li>use trigonometrical relationships (such as double-angle formulae) to facilitate the integration of functions such as <math>\cos^2 x</math></li> <li>integrate rational functions by means of decomposition into partial fractions (restricted to the types of partial fractions specified in paragraph 1 above)</li> <li>recognise an integrand of the form <math>\frac{kf(x)}{f(x)}</math>, and integrate, for example, <math>\frac{x}{x^2+1}</math> or <math>\tan x</math></li> <li>recognise when an integrand can usefully be regarded as a product, and use integration by parts to integrate, for example, <math>x \sin 2x</math>, <math>x^2 e^x</math> or <math>\ln x</math></li> <li>use a given substitution to simplify and evaluate either a definite or an indefinite integral</li> <li>use the trapezium rule to estimate the value of a definite integral, and use sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate.</li> </ul>

<p><b>6. Numerical solution of equations</b></p>	<ul style="list-style-type: none"> <li>• locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change</li> <li>• understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation</li> <li>• understand how a given simple iterative formula of the form <math>x_{n+1} = F(x_n)</math> relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy (knowledge of the condition for convergence is not included, but candidates should understand that an iteration may fail to converge).</li> </ul>
<p><b>7. Vectors</b></p>	<ul style="list-style-type: none"> <li>• understand the significance of all the symbols used when the equation of a straight line is expressed in the form <math>\mathbf{r} = \mathbf{a} + t\mathbf{b}</math></li> <li>• determine whether two lines are parallel, intersect or are skew</li> <li>• find the angle between two lines, and the point of intersection of two lines when it exists</li> <li>• understand the significance of all the symbols used when the equation of a plane is expressed in either of the forms <math>ax + by + cz = d</math> or <math>(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0</math></li> <li>• use equations of lines and planes to solve problems concerning distances, angles and intersections, and in particular: <ul style="list-style-type: none"> <li>– find the equation of a line or a plane, given sufficient information</li> <li>– determine whether a line lies in a plane, is parallel to a plane, or intersects a plane, and find the point of intersection of a line and a plane when it exists</li> <li>– find the line of intersection of two non-parallel planes</li> <li>– find the perpendicular distance from a point to a plane, and from a point to a line</li> <li>– find the angle between two planes, and the angle between a line and a plane.</li> </ul> </li> </ul>
<p><b>8. Differential equations</b></p>	<ul style="list-style-type: none"> <li>• formulate a simple statement involving a rate of change as a differential equation, including the introduction if necessary of a constant of proportionality</li> <li>• find by integration a general form of solution for a first order differential equation in which the variables are separable</li> <li>• use an initial condition to find a particular solution</li> <li>• interpret the solution of a differential equation in the context of a problem being modelled by the equation.</li> </ul>

## 9. Complex numbers

- understand the idea of a complex number, recall the meaning of the terms real part, imaginary part, modulus, argument, conjugate, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal
- carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in cartesian form  $x + iy$
- use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs
- represent complex numbers geometrically by means of an Argand diagram
- carry out operations of multiplication and division of two complex numbers expressed in polar form  $r(\cos \theta + i \sin \theta) \equiv r e^{i\theta}$
- find the two square roots of a complex number
- understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying and dividing two complex numbers
- illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram, e.g.  $|z - a| < k$ ,  $|z - a| = |z - b|$ ,  $\arg(z - a) = \alpha$ .

## Unit M1: Mechanics 1 (Paper 4)

Questions set will be mainly numerical, and will aim to test mechanical principles without involving difficult algebra or trigonometry. However, candidates should be familiar in particular with the following trigonometrical results:  $\sin^2 i + \cos^2 i = 1$ ,  $\sin 90^\circ - i = \cos i$ ,  $\cos 90^\circ - i = \sin i$ ,  $\tan i = \frac{\sin i}{\cos i}$ ,  $\sin^2 i + \cos^2 i = 1$ .

Vector notation will not be used in the question papers, but candidates may use vector methods in their solutions if they wish.

In the following content list, reference to the equilibrium or motion of a 'particle' is not intended to exclude questions that involve extended bodies in a 'realistic' context; however, it is to be understood that any such bodies are to be treated as particles for the purposes of the question.

Unit M1: Mechanics 1 (Paper 4)	
	Candidates should be able to:
<b>1. Forces and equilibrium</b>	<ul style="list-style-type: none"> <li>• identify the forces acting in a given situation</li> <li>• understand the vector nature of force, and find and use components and resultants</li> <li>• use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero or, equivalently, that the sum of the components in any direction is zero</li> <li>• understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component</li> <li>• use the model of a 'smooth' contact, and understand the limitations of this model</li> <li>• understand the concepts of limiting friction and limiting equilibrium; recall the definition of coefficient of friction, and use the relationship <math>F = \mu R</math> or <math>F \leq \mu R</math>, as appropriate</li> <li>• use Newton's third law.</li> </ul>
<b>2. Kinematics of motion in a straight line</b>	<ul style="list-style-type: none"> <li>• understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities (in one dimension only)</li> <li>• sketch and interpret displacement-time graphs and velocity-time graphs, and in particular appreciate that: <ul style="list-style-type: none"> <li>– the area under a velocity-time graph represents displacement</li> <li>– the gradient of a displacement-time graph represents velocity</li> <li>– the gradient of a velocity-time graph represents acceleration</li> </ul> </li> <li>• use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration (restricted to calculus within the scope of unit P1)</li> <li>• use appropriate formulae for motion with constant acceleration in a straight line.</li> </ul>

<p><b>3. Newton's laws of motion</b></p>	<ul style="list-style-type: none"> <li>• apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction</li> <li>• use the relationship between mass and weight</li> <li>• solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration</li> <li>• solve simple problems which may be modelled as the motion of two particles, connected by a light inextensible string which may pass over a fixed smooth peg or light pulley.</li> </ul>
<p><b>4. Energy, work and power</b></p>	<ul style="list-style-type: none"> <li>• understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force (use of the scalar product is not required)</li> <li>• understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae</li> <li>• understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy</li> <li>• use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion</li> <li>• solve problems involving, for example, the instantaneous acceleration of a car moving on a hill with resistance.</li> </ul>

## Unit M2: Mechanics 2 (Paper 5)

Knowledge of the content of unit M1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

	<i>Candidates should be able to:</i>
<b>1. Motion of a projectile</b>	<ul style="list-style-type: none"><li>• model the motion of a projectile as a particle moving with constant acceleration and understand any limitations of the model</li><li>• use horizontal and vertical equations of motion to solve problems on the motion of projectiles, including finding the magnitude and direction of the velocity at a given time or position, the range on a horizontal plane and the greatest height reached</li><li>• derive and use the cartesian equations of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown.</li></ul>
<b>2. Equilibrium of a rigid body</b>	<ul style="list-style-type: none"><li>• calculate the moment of a force about a point, in two dimensional situations only (understanding of the vector nature of moments is not required)</li><li>• use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body, and identify the position of the centre of mass of a uniform body using considerations of symmetry</li><li>• use given information about the position of the centre of mass of a triangular lamina and other simple shapes</li><li>• determine the position of the centre of mass of a composite body by considering an equivalent system of particles (in simple cases only, e.g. a uniform L-shaped lamina)</li><li>• use the principle that if a rigid body is in equilibrium under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this</li><li>• solve problems involving the equilibrium of a single rigid body under the action of coplanar forces, including those involving toppling or sliding (problems set will not involve complicated trigonometry).</li></ul>
<b>3. Uniform motion in a circle</b>	<ul style="list-style-type: none"><li>• understand the concept of angular speed for a particle moving in a circle, and use the relation <math>v = r\omega</math></li><li>• understand that the acceleration of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae <math>r\omega^2</math> and <math>\frac{v^2}{r}</math></li><li>• solve problems which can be modelled by the motion of a particle moving in a horizontal circle with constant speed.</li></ul>

<p><b>4. Hooke's law</b></p>	<ul style="list-style-type: none"> <li>• use Hooke's law as a model relating the force in an elastic string or spring to the extension or compression, and understand the term modulus of elasticity</li> <li>• use the formula for the elastic potential energy stored in a string or spring</li> <li>• solve problems involving forces due to elastic strings or springs, including those where considerations of work and energy are needed.</li> </ul>
<p><b>5. Linear motion</b></p> <p><b>under a variable</b></p> <p><b>force</b></p>	<ul style="list-style-type: none"> <li>• use <math>\frac{dx}{dt}</math> for velocity, and <math>\frac{dv}{dt}</math> or <math>v\frac{dv}{dx}</math> for acceleration, as appropriate</li> <li>• solve problems which can be modelled as the linear motion of a particle under the action of a variable force, by setting up and solving an appropriate differential equation (restricted to equations in which the variables are separable).</li> </ul>

**Unit S1: Probability & Statistics 1 (Paper 6)**

	<i>Candidates should be able to:</i>
<b>1. Representation of data</b>	<ul style="list-style-type: none"> <li>• select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have</li> <li>• construct and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs</li> <li>• understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation), e.g. in comparing and contrasting sets of data</li> <li>• use a cumulative frequency graph to estimate the median value, the quartiles and the interquartile range of a set of data</li> <li>• calculate the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals such as <math>\Sigma x</math> and <math>\Sigma x^2</math>, or <math>\Sigma(x - a)</math> and <math>\Sigma(x - a)^2</math>.</li> </ul>
<b>2. Permutations and combinations</b>	<ul style="list-style-type: none"> <li>• understand the terms permutation and combination, and solve simple problems involving selections</li> <li>• solve problems about arrangements of objects in a line, including those involving: <ul style="list-style-type: none"> <li>– repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS')</li> <li>– restriction (e.g. the number of ways several people can stand in a line if 2 particular people must – or must not – stand next to each other).</li> </ul> </li> </ul>
<b>3. Probability</b>	<ul style="list-style-type: none"> <li>• evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events (e.g. for the total score when two fair dice are thrown), or by calculation using permutations or combinations</li> <li>• use addition and multiplication of probabilities, as appropriate, in simple cases</li> <li>• understand the meaning of exclusive and independent events, and calculate and use conditional probabilities in simple cases, e.g. situations that can be represented by means of a tree diagram.</li> </ul>
<b>4. Discrete random variables</b>	<ul style="list-style-type: none"> <li>• construct a probability distribution table relating to a given situation involving a discrete random variable <math>X</math>, and calculate <math>E(X)</math> and <math>\text{Var}(X)</math></li> <li>• use formulae for probabilities for the binomial distribution, and recognise practical situations where the binomial distribution is a suitable model (the notation <math>B(n, p)</math> is included)</li> <li>• use formulae for the expectation and variance of the binomial distribution.</li> </ul>

**5. The normal distribution**

- understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables
- solve problems concerning a variable  $X$ , where  $X \sim N(\mu, \sigma^2)$ , including:
  - finding the value of  $P(X > x_1)$ , or a related probability, given the values of  $x_1, \mu, \sigma$
  - finding a relationship between  $x_1, \mu$  and  $\sigma$  given the value of  $P(X > x_1)$  or a related probability
- recall conditions under which the normal distribution can be used as an approximation to the binomial distribution ( $n$  large enough to ensure that  $np > 5$  and  $nq > 5$ ), and use this approximation, with a continuity correction, in solving problems.

**Unit S2: Probability & Statistics 2 (Paper 7)**

Knowledge of the content of unit S1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

	<i>Candidates should be able to:</i>
<b>1. The Poisson distribution</b>	<ul style="list-style-type: none"><li>• calculate probabilities for the distribution <math>Po(n)</math></li><li>• use the fact that if <math>X \sim Po(n)</math> then the mean and variance of <math>X</math> are each equal to <math>n</math></li><li>• understand the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model</li><li>• use the Poisson distribution as an approximation to the binomial distribution where appropriate (<math>n &gt; 50</math> and <math>np &lt; 5</math>, approximately)</li><li>• use the normal distribution, with continuity correction, as an approximation to the Poisson distribution where appropriate (<math>n &gt; 15</math>, approximately).</li></ul>
<b>2. Linear combinations of random variables</b>	<ul style="list-style-type: none"><li>• use, in the course of solving problems, the results that:<ul style="list-style-type: none"><li>– <math>E(aX + b) = aE(X) + b</math> and <math>Var(aX + b) = a^2Var(X)</math></li><li>– <math>E(aX + bY) = aE(X) + bE(Y)</math></li><li>– <math>Var(aX + bY) = a^2Var(X) + b^2Var(Y)</math> for independent <math>X</math> and <math>Y</math></li><li>– if <math>X</math> has a normal distribution then so does <math>aX + b</math></li><li>– if <math>X</math> and <math>Y</math> have independent normal distributions then <math>aX + bY</math> has a normal distribution</li><li>– if <math>X</math> and <math>Y</math> have independent Poisson distributions then <math>X + Y</math> has a Poisson distribution.</li></ul></li></ul>
<b>3. Continuous random variables</b>	<ul style="list-style-type: none"><li>• understand the concept of a continuous random variable, and recall and use properties of a probability density function (restricted to functions defined over a single interval)</li><li>• use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution (explicit knowledge of the cumulative distribution function is not included, but location of the median, for example, in simple cases by direct consideration of an area may be required).</li></ul>

<p><b>4. Sampling and estimation</b></p>	<ul style="list-style-type: none"> <li>• understand the distinction between a sample and a population, and appreciate the necessity for randomness in choosing samples</li> <li>• explain in simple terms why a given sampling method may be unsatisfactory (knowledge of particular sampling methods, such as quota or stratified sampling, is not required, but candidates should have an elementary understanding of the use of random numbers in producing random samples)</li> <li>• recognise that a sample mean can be regarded as a random variable, and use the facts that <math>E^{\wedge} \bar{X}_h = \mu</math> and that <math>\text{Var}^{\wedge} \bar{X}_h = \frac{\sigma^2}{n}</math></li> <li>• use the fact that <math>\bar{X}</math> has a normal distribution if <math>X</math> has a normal distribution</li> <li>• use the Central Limit Theorem where appropriate</li> <li>• calculate unbiased estimates of the population mean and variance from a sample, using either raw or summarised data (only a simple understanding of the term 'unbiased' is required)</li> <li>• determine a confidence interval for a population mean in cases where the population is normally distributed with known variance or where a large sample is used</li> <li>• determine, from a large sample, an approximate confidence interval for a population proportion.</li> </ul>
<p><b>5. Hypothesis tests</b></p>	<ul style="list-style-type: none"> <li>• understand the nature of a hypothesis test, the difference between one-tail and two-tail tests, and the terms null hypothesis, alternative hypothesis, significance level, rejection region (or critical region), acceptance region and test statistic</li> <li>• formulate hypotheses and carry out a hypothesis test in the context of a single observation from a population which has a binomial or Poisson distribution, using either direct evaluation of probabilities or a normal approximation, as appropriate</li> <li>• formulate hypotheses and carry out a hypothesis test concerning the population mean in cases where the population is normally distributed with known variance or where a large sample is used</li> <li>• understand the terms Type I error and Type II error in relation to hypothesis tests</li> <li>• calculate the probabilities of making Type I and Type II errors in specific situations involving tests based on a normal distribution or direct evaluation of binomial or Poisson probabilities.</li> </ul>

